Verification and Probabilistic Logic Programming

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Model Checking

$$S \overset{?}{\models} \varphi$$

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- S: System specification or implementation
  Automaton, transition system, protocol specification, process expression, code, ...
- φ: Property specification
  Temporal logic formula

Verification of other forms (e.g. refinement checking) are not considered in this talk.

Executable Specifications and Logic Programs

Logic Programming is well-recognized for its suitability for

- Writing interpreters for languages starting from high-level declarative specifications
- Constructing state spaces and searching through them
- Performing meaning-preserving abstractions using clever data representations

Executable Specification of Operational Semantics

$$\frac{e_1 \to e_1'}{(e_1\ e_2) \to (e_1'\ e_2)}$$

```
step(app(E1, E2), app(E1P, E2)) :-
    step(E1, E1P).
```

$$\frac{e_2 \to e_2'}{(v_1\ e_2) \to (v_1\ e_2')}$$

```
step(app(V1, E2), app(V1, E2P)) :-
    isValue(V1),
    step(E2, E2P).
```

$$(\lambda x.\ e_1)\ \ v_2 \to [x \mapsto v_2]e_1$$

```
step(app(lambda(X, E1), V2), E2) :-
    isValue(V2),
    subst(X, V2, E1, E2).

isValue(lambda(_, _)).
```

[Call-By-Value Lambda Calculus]

Substitution

$$\begin{array}{lcll}
[x \mapsto s]x & = & s & \\
[x \mapsto s]y & = & y & \text{if } y \neq x \\
[x \mapsto s](\lambda y.\ t) & = & \lambda y.\ [x \mapsto s]t & \text{if } x \neq y \text{ and } y \notin fv(s) \\
[x \mapsto s](t_1\ t_2) & = & ([x \mapsto s]t_1)\ ([x \mapsto s]t_2) &
\end{array}$$

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- We can program α-renaming explicitly, or better still. . .

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\end{array}$$

- This definition becomes complete only when we consider α-renaming.
- We can program α-renaming explicitly, or better still…
- With suitable restrictions on the way λ-terms are written,
  - represent variables in lambda-terms with logical variables, and
  - use the "standardization" done by resolution to perform the needed α-renaming.
- We used such a strategy to encode model checkers for the *pi*-calculus [Yang et al, VMCAI'03].

Executable Specification of Abstract Semantics

$$\frac{\texttt{p = \&q}}{\texttt{p} \to \texttt{q}}$$

$$\frac{\texttt{p = q} \quad \texttt{q} \to r}{\texttt{p} \to r}$$

$$\frac{\texttt{p = *q} \quad \texttt{q} \to r \quad r \to s}{\texttt{p} \to s}$$

$$\frac{\texttt{*p = q} \quad \texttt{p} \to r \quad \texttt{q} \to s}{r \to s}$$

```
pts(P,Q) :-
    stmt(v(P), addr(Q)).

pts(P,R) :-
    stmt(v(P), v(Q)),
    pts(Q, R).

pts(P,S) :-
    stmt(v(P), star(Q)),
    pts(Q, R), pts(R, S).

pts(R, S) :-
    stmt(star(P), v(Q)),
    pts(P, R),
    pts(Q, S).
```

[Anderson's Context-Insensitive Points-To Analysis]

Demand-Driven Analysis

Compute only the information necessary to determine the *may-point-to* set of x. [Heinze et al., PLDI 2001]

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Demand-Driven Analysis

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- Tabled query evaluation is naturally demand-driven, but …
- Clauses of the form `pts(R, S) :- stmt(star(P), v(Q)), ...` lead to generate-and-test evaluation.
- **Trick:** replicate *points-to* (`pts`) as *pointed-to-by* (`ptb`).

```
pts(R, S) :-
    stmt(star(P), v(Q)),
    pts(P, R),
    pts(Q, S).
```

⇒

```
pts(R, S) :-
    ptb(R, P),
    stmt(star(P), v(Q)),
    pts(Q, S).
```

[PPDP'05]

Model Checking as Query Evaluation

- Encode the semantic equations of temporal logics as a logic program.
- Query evaluation over the program will perform model checking.

We consider traditional query evaluation methods developed and used in LP literature.

- Rybalchenko et al take a very different (and neat) approach: posing verification problems as constraint solving over Horn Constraints.
- Verification of certain infinite-state systems is enabled by the construction and use of specialized Horn Constraint solvers.

Executable Specification of Semantic Equations

$[\![\cdot]\!]$ is the smallest set such that:

% $[\![p]\!]$ = states satisfying prop. p.
$[\![p]\!] = \{s \mid p \in AP(s)\}$

% Conjunction:
$[\![\varphi_1 \wedge \varphi_2]\!] = [\![\varphi_1]\!] \cap [\![\varphi_2]\!]$

% $[\![\mathrm{EF}\ f]\!] =$
% $\quad \{s \mid \exists t.\ \ s \xrightarrow{*} t \text{ and } t \in [\![f]\!]\}$

$$[\![EF\varphi]\!] = [\![\varphi]\!] \cup \{s \mid \exists t.\ s \rightarrow t, t \in [\![EF\varphi]\!]\}$$

$\vdots$

```
models(S,prop(P)) :-
    holds(S, P).

models(S,and(F1,F2)) :-
    models(S, F1), models(S, F2).

models(S, ef(F)) :-
    models(S, F).
models(S, ef(F)) :-
    trans(S, T), models(T, ef(F)).

models(S, af(F)) :-
    models(S, F).
models(S, af(F)) :-
    findall(T, trans(S, T), L),
    all_models(T, af(F)).
...
```

[Computation Tree Logic's Semantics (Fragment)]

Model Checking as Query Evaluation

Mobile Ad-Hoc Networks

Parameterized Systems

Multi-Agent Systems

Model Checkers

`Infinite-State Systems`

π-Calculus

Model Checking as Query Evaluation

Mobile Ad-Hoc Networks

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Probabilistic Systems

System Models: Kripke Structures and LTSs

- State transition systems: *directed graphs* with
  - Vertices representing system states, and
  - Edges representing transitions between states
- Labels on edges representing "actions": *Labeled Transition Systems*.
- Vertices associated with sets of propositions: *Kripke structures*.

Property Specification: Temporal Logics I

Computational Tree Logic (CTL):

φ	$\rightarrow$	$p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi$	Propositions, logical connectives
	$\mid$	$E\phi \mid A\phi$	State formulae
ϕ	$\rightarrow$	$X\varphi \mid \varphi_1 \; U \; \varphi_2 \mid \varphi_1 R \; \varphi_2$	Path formulae

- Semantics of the CTL is usually given in terms of computation trees of Kripke Structures.
- The meaning of path formulae are given in terms of sets of paths (runs, or sequences of states) of the system.
- Informally, $\varphi_1 \; U \; \varphi_2$ holds in a run means φ_1 holds in every state of the run until a state where φ_2 holds.
- *Derived* path formulae $F \; \varphi$ and $G \; \varphi$ are often used for simplicity. $F \; \varphi \equiv tt \; U \; \varphi$

Property Specification: Temporal Logics II

Let K be a Kripke structure and $AP(s)$ denote the set of atomic propositions associated with state s in K.

- $K, s \models p$ if $p \in AP(s)$
- $K, s \models \varphi_1 \wedge \varphi_2$ if $K, s \models \varphi_1$ and $K, s \models \varphi_2$. (sim. for "$\vee$")
- $K, s \models A\phi$ if for *every* path π in K with $init(\pi) = s$, $K, \pi \models \phi$.
- $K, s \models E\phi$ if for *some* path π in K with $init(\pi) = s$, $K, \pi \models \phi$.
- $K, \pi \models X\varphi$ if $K, \pi[1] \models \varphi$
- $K, \pi \models \varphi_1 \; U \; \varphi_2$ if $\pi = s_1, s_2, \ldots$, and $\exists i \geq 1$ such that
  - $K, s_i \models \varphi_2$, and
  - $\forall 1 \leq j < i$, $K, s_j \models \varphi_1$.
- $\ldots$

Property Specification: Temporal Logics III

Modal Mu-Calculus

φ	$\rightarrow$	$tt \mid \varphi \wedge \varphi \mid \varphi \vee \varphi$	logical connectives
	$\mid$	$\langle \alpha \rangle \varphi$	Diamond formulae
	$\mid$	$[\alpha]\varphi$	Box formulae
	$\mid$	X	Formula variable
	$\mid$	$\mu X.\varphi$	Least fixed point formula
	$\mid$	$\nu X.\varphi$	Greatest fixed point formula

Semantics of modal mu-calculus formulae are defined over LTSs, with α ranging over the actions of the LTS.

Property Specification: Temporal Logics IV

Examples in the “assembly language of propositional temporal logics”:

1. “a” action is eventually possible:

$$\mu X.(\langle a\rangle tt) \vee (\langle - \rangle X)$$

2. “b” action is eventually possible *from each state*:

$$\nu Y.[-]Y \wedge (\mu X.(\langle b\rangle tt) \vee (\langle - \rangle X))$$

3. “c” action is enabled infinitely often on all infinite paths:

$$\nu Y.\mu X.[-](((\langle c\rangle tt \wedge Y) \vee X)$$

System Specification: Process Languages

- Core languages such as CCS, pi-calculus etc. that elucidate the meaning of interleaving, synchronization, communication etc.
- Value-passing languages add variables and values to the core to allow succinct specifications.
- Meanings of specifications in such languages are given in terms of LTSs and Kripke Structures ("translating them down").
- Derived LTSs may not be finite state.
- Richer languages permit description of
    - Push-down systems (analogous to recursive programs)
    - Parameterized systems (where aspects of a systems, e.g. number of processes of a specific family, may be only symbolically specified).

A Model Checker for CTL: I

CTL formulae represented as Prolog Terms of the following forms:

- `prop`(P) atomic propositions
- `neg`(F), `and`(F_1, F_2), `or`(F_1, F_2): logical connectives.
- `ex`(F), `eu`(F_1, F_2), `er`(F_1, F_2): formulae with existential path quantifier
- `ax`(F), `au`(F_1, F_2), `ar`(F_1, F_2): formulae with universal path quantifier

Kripke structure represented as a set of Prolog facts:

- `trans`(S,T): transition from state S to state T.
- `holds`(S,P): proposition P holds at state S.

A Model Checker for CTL: II

```
% Propositions and their negations
models(S, prop(P)) :- holds(S, P).
models(S, neg(prop(P))) :- not holds(S, P).

% Conjunction and Disjunction
models(S, and(F1,F2)) :- models(S, F1), models(S, F2).
models(S, or(F1, F2)) :- models(S, F1); models(S, F2).

% EX:
models(S, ex(F)) :- trans(S, T), models(T, F).
```

A Model Checker for CTL: II

- $E\varphi_1\ U\ \varphi_2$ can be “unrolled” as:

$$\varphi_2 \vee (\varphi_1 \wedge (EX\ (E\varphi_1 U \varphi_2)))$$

  where the unrolling is finite (least fixed point).

- Hence:

```
% EU
models(S, eu(F1, F2)) :-
    models(S, or(F2, and(F1, ex(eu(F1, F2))))).
```

- Similarly:

```
% AU
models(S, au(F1, F2)) :-
    models(S, or(F2, and(F1, ax(au(F1, F2))))).
```

A Model Checker for CTL: III

- $A\phi \equiv \neg E \neg \phi$ and $\neg X \varphi \equiv X \neg \varphi$
- But encoding `ax` in terms of negation and `ex` means our unrolling of `au` will result in a non-stratified program.
- Hence:

```
% AX
models(S, ax(F)) :-
    findall(T, trans(S, T), L),
    all_models(L, F).
```

where

```
all_models([], F).
all_models([S|L], F) :- models(S, F), all_models(L, F).
```

A Model Checker for CTL: IV

- $\neg(\varphi_1 R\ \varphi_2) \equiv (\neg\varphi_1) U\ (\neg\varphi_2)$
- Hence:

```
% ER
models(S, er(F1, F2)) :-
    negate(F1, NF1),
    negate(F2, NF2),
    tnot models(S, au(NF1, NF2)).
```

where `negate`(F, NF) binds NF to the negation normal form of `neg`(F).

Tabled Evaluation for Model Checking

- Tabled resolution is needed for termination
  - Note unrolling of `eu` and `au`
- `models/2` is not statically stratified
  - Note use of negative dependency in treatment of `er`.
- But query evaluation will be dynamically stratified
  - Expansion of `er` using `au` is not unrolling, and does not lead to cycles.

Complexity of CTL Model Checking

- `models/2`: Given a Kripke structure with $|S|$ states, and a formula of size $|\varphi|$, there are at most $O(|S|.|\varphi|)$ distinct calls to `models/2`.
- Each call is *ground*, so as at most one answer.
- If $|T|$ is the size of the Kripke structure (max. of number of states and transitions), then query evaluation takes $O(|T|.|\varphi|)$ steps.
- Access into the call tables may take an additional $O(|\varphi|)$ time per access.
- But with hash-consing (or any other suitable representation of a formula term), table access time will be $O(1)$ with perfect indexing.
- Hence model checking can be done in $O(|T|.|\varphi|)$ time and $(|S|.|\varphi|)$ space.

Model Checker for the Modal Mu-Calculus: I

Equational form of the modal mu-calculus.

- $\mu X.(\langle a\rangle tt) \vee (\langle - \rangle X)$
  written as $x \overset{\mu}{=} (\langle a\rangle tt) \vee (\langle - \rangle x)$
- $\nu Y.[-]Y \wedge (\mu X.(\langle b\rangle tt) \vee (\langle - \rangle X))$
  written as a set of two equations:

$$\begin{aligned} y &\overset{\nu}{=} [-]y \wedge x \\ x &\overset{\mu}{=} (\langle b\rangle tt) \vee (\langle - \rangle x) \end{aligned}$$

- $\nu Y.\mu X.[-]((\langle c\rangle tt \wedge Y) \vee X)$
  written as a set of two parameterized equations:

$$\begin{aligned} y &\overset{\nu}{=} x(y) \\ x(Z) &\overset{\mu}{=} [-]((\langle c\rangle tt \wedge Z) \vee x(Z)) \end{aligned}$$

Model Checker for the Modal Mu-Calculus: II

- Assume equational form of mu-calculus formulas are represented by a set of facts. The RHS of equations are represented a Prolog terms.

```
% Propositions and their negations
models(S, tt).
models(S, and(F1,F2)) :- models(S, F1), models(S, F2).
models(S, or(F1, F2)) :- models(S, F1); models(S, F2).

% Diamond:
models(S,diam(A,F)) :- trans(S, A, T), models(T, F).

% Box :
models(S, box(A,F)) :-
    findall(T, trans(S, A, T), L), all_models(L, F).
```

- Note the action label “A” in the transition relation.

Model Checker for the Modal Mu-Calculus: trial

- Assume equational form of mu-calculus formulas are represented by a set of facts of the form
  - `lfp`(x, φ) for μ equations, and
  - `gfp`(x, φ) for ν equations.

```
% LFP formula
models(S, form(X)) :- lfp(X, F), models(S, F).

% GFP formula
models(S, form(X)) :-
    gfp(X, F),
    negate(F, NF),
    tnot models(S, NF).
```

Alternation Freedom and Stratification

- As in the case of CTL model checker, the `models/2` predicate defining the mu-calculus model checker is not statically stratified.
- But for formulae with a single fixed point, or alternation-free fixed points, query evaluation is dynamically stratified.
- For formulae with alternation, query evaluation may not even be dynamically stratified.
    - One strategy is to generate a “residual” program that retains the cycles through negation, and generate a preferred stable model of the residue.

Complexity of Modal Mu-Calculus Model Checking

Time and space complexity of query evaluation over `models/2` can be analyzed along the same lines as used for CTL.

- For an alternation-free formula φ, model checking can be done in
  - $O(|S|.|\varphi|)$ space, where $|S|$ is the number of states in the LTS.
  - $O(|T|.|\varphi|)$ time, where $|T|$ is the size of the LTS.

Beyond Finite-State Model Checking: I

- Use constraints to represent sets of equivalent states.
    - Finite number of equivalence classes implies termination (e.g. classical timed automata).
- Allow data variables in *Property Specification* that may unify with data fields in system specification.
    - Encoding is agnostic to which side has variables.
    - Enables verification of a class of *data independent* systems.

Beyond Finite-State Model Checking: II

- Use Forall-Exists quantified Horn Clauses as a constraint language with powerful (albeit incomplete) solvers.
- A number of model checking problems, including CTL model checking of infinite state systems, can be cast as a satisfaction problem over the above constraint language.

Logic Programs

Program Rules

$+$ $\models$ **Query Answers**

Facts

Probabilistic Logic Programs

Program Rules

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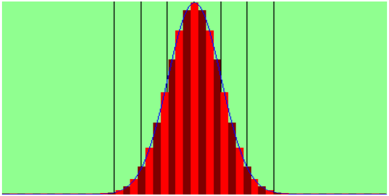
`Probabilistic Facts`



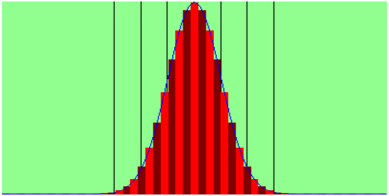
ICL, PRISM, ProbLog, . . .

Probabilistic Logic Programs

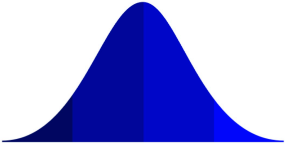
Program Rules

$+$

Probabilistic Facts



$\models$ **Query Answers**



ICL, PRISM, ProbLog, . . .

Independent Choice Logic (ICL)

Poole's ICL forms a precursor to modern PLP languages.
An ICL theory has:

- An *acyclic* logic program
- A collection of atom sets, e.g. $\{\{a_0, a_1\}, \{b_0, b_1, b_2\}\}$, called the choice space.
    - Each set in the collection can be viewed as a random variable; each atom in the set is a possible outcome of the variable.
- Distributions over the choice space.

Semantics of ICL given in terms of the distributions over the choice space.

Stochastic Logic Programs (SLP)

- SLP [Muggleton et al] defines a probability distribution over program clauses.
- Probability of a query answer is computed based on the probabilities of clauses used during resolution.
- SLP is expressive enough to represent a large class of non-recursive stochastic systems (e.g. non-recursive Stochastic Context Free Grammars).

PRISM

A language for probabilistic logic programming with system for inference and parameter learning (Sato et al, since '99).

- Logic programs with a set of **probabilistic facts**: `msw(X, I, V)`, where
    - `X` is a discrete-valued random process
    - `V` is a value generated by the random process
    - `I` is the *instance number*, distinguishing different trials.
- Random variables generated by the same random process are i.i.d.
- Random variables generated by distinct random processes are independent.
- Has a well-defined model-theoretic (*distribution*) semantics, and an operational semantics based on tabled resolution.

Distribution semantics

```
% "a" is a boolean random process
p(X) :-  msw(a, 0, X),
         msw(a, 1, Y),
         X=Y.
values(a, [t,f]).
set_sw(a, [0.3,0.7])
```

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Worlds:

msw(a,0,t) msw(a,1,t)	msw(a,0,t) msw(a,1,f)
msw(a,0,f) msw(a,1,t)	msw(a,0,f) msw(a,1,f)

- Outcomes of random processes define worlds.

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Worlds:

msw(a,0,t) msw(a,1,t) 0.09	msw(a,0,t) msw(a,1,f) 0.21
msw(a,0,f) msw(a,1,t) 0.21	msw(a,0,f) msw(a,1,f) 0.49

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- In each world, `msw`s form a set of logical (non-probabilistic) facts.

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Models:

msw(a,0,t) msw(a,1,t) 0.09 p(t)	msw(a,0,t) msw(a,1,f) 0.21
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- Outcomes of random processes define worlds.
- The probability of a world is assigned based on the probabilities of the outcomes in the world.
- In each world, `msw`s form a set of logical (non-probabilistic) facts.
- Distribution over least models: the least model in each world is assigned the probability of that world.

ProbLog

- At its simplest, a ProbLog program resembles a Prolog program where each clauses is annotated with a discrete probability value.
- These annotations define a distribution of (non-probabilistic) programs, resulting in a distribution semantics.
- ProbLog and PRISM program, when restricted to discrete distributions, can be translated to one another, with the same distribution semantics.
- ProbLog query evaluation materializes explanations in non-trivial structures, and is not subject to PRISM-style independence and mutual exclusion restrictions.

Probabilistic Logic Programs: A quick tour

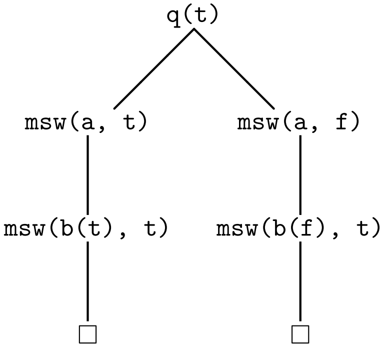
- Logic-based representation of statistical models
    - Examples include BLPs (Kersting and De Raedt, '00), PRMs (Friedman et al, '99), MLNs (Richarson and Domingos, '06).
    - The underlying statistical network, derived from logical/statistical specifications, is finite.
- Statistical inference over proof structures
    - Conservative extension to traditional logic programs, with explicit or implicit use of random variables and processes.
    - Examples include PRISM (Sato and Kameya, '99), ICL (Poole, '93), CLP(BN) (Santos Costa et al, '03), ProbLog (De Raedt et al, '07), LPAD (Vennekens et al, '09).
    - In terms of expressive power, PRISM, ProbLog and LPAD coincide; however, they use different inference procedures.

Evaluation in PRISM — I

```
% Finite Mixture Model
q(Y) :-  msw(a, 0, X),
          msw(b(X), 0, Y).

values(a, [t,f]).
values(b(_), [t,f]).
set_sw(a, [0.3,0.7])
set_sw(b(t), [0.6,0.4])
set_sw(b(f), [0.5,0.5])
```

Explanations



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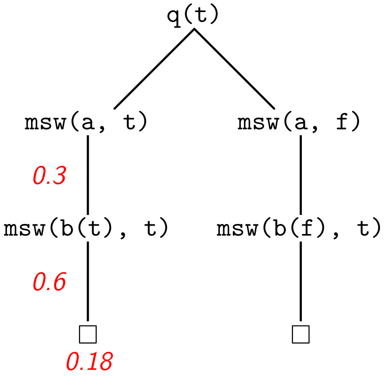
Explanations *and Probabilities*



Evaluation in PRISM — I

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% Finite Mixture Model
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          msw(b(X), 0, Y).

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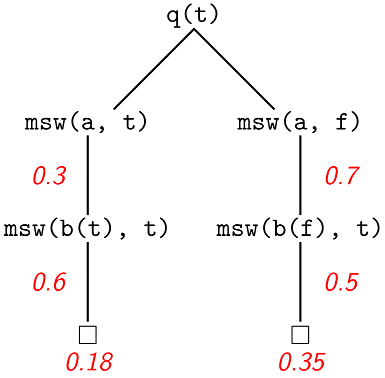

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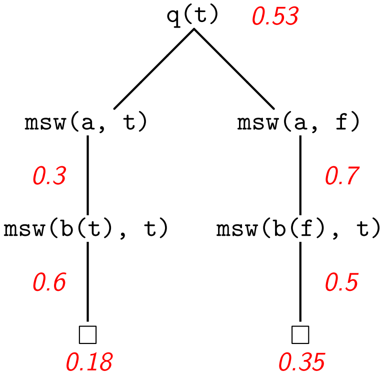

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- The probability of an answer is the probability of the set of explanations of the answer.
  - If explanations are pairwise mutually exclusive, then the probability of the set of explanations is the sum of probabilities of each explanation. [Mutual Exclusion assumption]
  - If the set of explanations is finite, then this sum can be effectively computed. [Finiteness assumption]

Generalizations

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    - Inference mimics the best known algorithms for certain statistical models (e.g. Viterbi alg. for HMMs).
- ProbLog and PITA (an implementation of LPAD) use BDDs to represent the set of explanations, and consequently remove Independence and Mutual Exclusion assumptions.
    - Finiteness assumption is still needed since the BDDs need to be effectively constructed.

Evaluation via Knowledge Compilation: I

- Explanations can be viewed as residues of partially evaluating all but the probabilistic facts.
- Each set of explanations can be mapped to a Boolean propositional formula.
  - Each explanation is a set (conjunction) of random variable valuations.
  - Each explanation in an explanation set (i.e. disjunction) supports the derived answer.
- Explanations can be materialized more succinctly using other Boolean formula representations, such as Deterministic Disjunctive Negation Normal Forms (dDNNFs), etc.

Evaluation via Knowledge Compilation: II

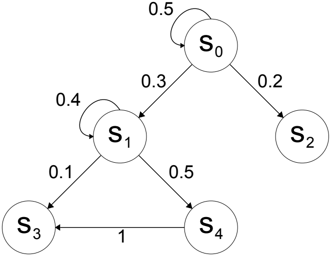
- ProbLog2 uses the more succinct representation of explanations.
- *Weighted model count* is a measure to each Boolean formula, defined a weighted sum of (the number of) satisfiable assignments.
- Weighted model counting can be done in time polynomial in the size of a Boolean formula's dDNNF representation.

Finiteness Assumption and PRISM

- In ICLP'12, Sato and Meyer present a method to generate equations from probabilistic programs with loops.
- This mechanism essentially hides the "instance" variable in `msws`.
- It proceeds under the assumption that different occurrences along a single explanation are independent.
    - This assumption holds for individual runs of a Markov Chain or prefix probability computations in Probabilistic CFGs, but does not satisfied in general.

Probabilistic Transition Systems in PRISM

Example Markov Chain



Probabilistic Transition Systems in PRISM

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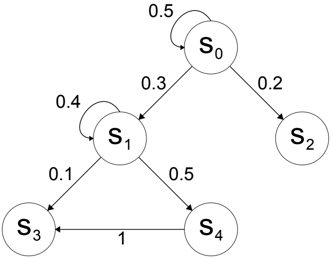


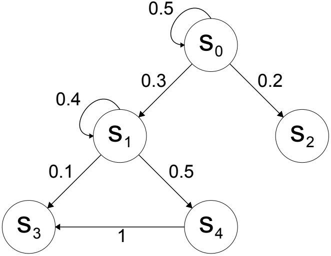
```
% Encoding as a Probabilistic LP
trans(S, I, T) :- msw(t(S), I, T).

% Ranges
:- values(t(s0), [s0, s1, s2]).
:- values(t(s1), [s1, s3, s4]).
:- values(t(s4), [s3]).

% Distributions
set_sw(t(s0), [0.5, 0.3, 0.2]).
set_sw(t(s1), [0.4, 0.1, 0.5]).
set_sw(t(s4), [1]).
```

Probabilistic Transition Systems in PRISM

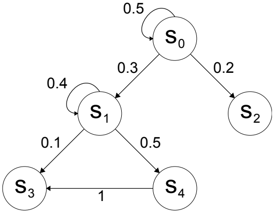
Example Markov Chain



```
% Encoding as a Probabilistic LP
trans(S, I, T) :- msw(t(S), I, T).

% Encoding of Reachability
reach(S, I, T) :-
    trans(S, I, U),
    reach(U, next(I), T).
reach(S, _, S).
```

Probabilistic Model Checking as Query Evaluation



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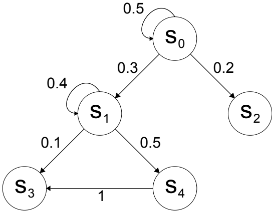

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- `|?- prob(reach(`s_0`, 0, `s_3`)).`

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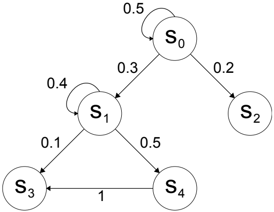

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- Evaluation of the above query will not terminate!

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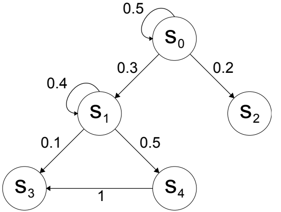

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    - There are infinitely many *explanations* for `reach(`s_0`, 0, `s_3`)`

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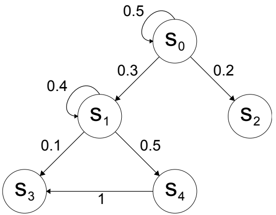

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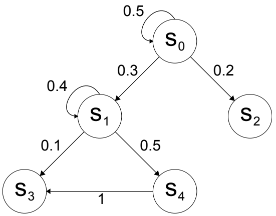

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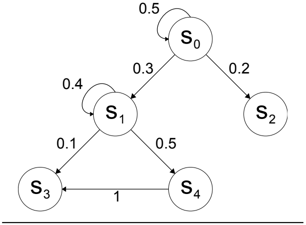
    - There are infinitely many *explanations* for `reach(`s_0`, 0, `s_3`)`
- Distribution semantics is well-defined and gives the correct probability, but standard inference methods cannot evaluate this query.
- More recent extension in PRISM removes finiteness assumption under restricted conditions.

Explanations



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Explanations for `reach(s0,0,s3)`:

- `msw`(t(s0), 0, s1), `msw`(t(s1), *next*(0), s3).
- `msw`(t(s0), 0, s0), `msw`(t(s0), *next*(0), s1), `msw`(t(s1), *next*(*next*(0)), s3).

⋮

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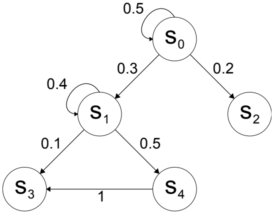

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Note: `prob(reach(s0,0,s3))` is same as `prob(reach(s0,H,s3))` for any `H`.

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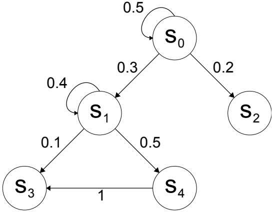

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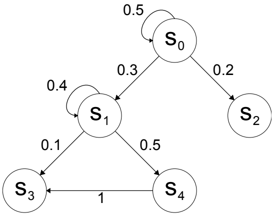

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We can use a *grammar* to represent the set of explanations for the abstracted query.

$$\texttt{expl}(\texttt{reach}(s0, H, s3)) \longrightarrow$$
$$[\texttt{msw}(t(s0), H, s0)],$$
$$\texttt{expl}(\texttt{reach}(s0, next(H), s3)).$$
$$\texttt{expl}(\texttt{reach}(s0, H, s3)) \longrightarrow$$
$$[\texttt{msw}(t(s0), H, s1)],$$
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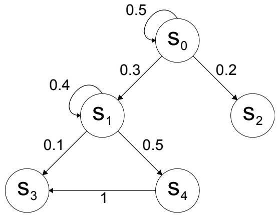

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&\quad \mathtt{expl}(\mathtt{reach}(s1, next(H), s3)).
\end{aligned}
$$

is similar to the stochastic grammar:

$$S_0 \xrightarrow{0.5} S_0$$

$$S_0 \xrightarrow{0.3} S_1$$

Answer probability of `reach(s0,H,s3)` is the *language probability* of the above SCFG

and is the least solution to equations of the form:

$x_0 = 0.5x_0 + 0.3x_1$

Abstraction for temporal programs

- *Instance* and *Non-Instance* variables belong to distinct sorts. Variables of one sort cannot be unified with those of the other.
- Only terms containing instance variables can be used as instance arguments of `msw`.
- For any clause of a predicate with an instance argument, the instances on the LHS of the clause must be a subterm of instances on the RHS. This imposes an ordering on time.

Temporally Well-Formed Programs

- A probabilistic logic program with annotations of the form `temporal`$(p/n-i)$.
  - Example: `temporal(reach/3-2)`
  - `reach` is a *temporal* predicate
  - The second argument of an atom with root `reach` is its *instance argument*.
- For a rule defining a temporal predicate, the instance argument of the head must be a subterm of instance arguments of every temporal body predicate.
  - Example:

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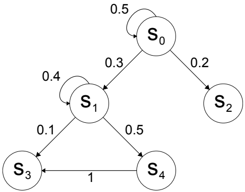
- Instance arguments are not bound to non-instance arguments, or vice versa.
- In explanation grammars of temporally well-formed programs, `msw`(r, t, x) will always be independent of any `msw` derived from non-terminal `expl(p)`
    - if t is a proper subterm of p's instance argument.

Factored Equation Diagrams

Not all explanation grammars can be translated directly to stochastic grammars.



- Consider the query

  `reach(s0, H, s3); reach(s0, H, s4).`

- The grammar will have productions of the form:

$$\mathtt{expl}(\mathtt{reach}(s0, H, s3); \mathtt{reach}(s0, H, s4)) \longrightarrow \mathtt{expl}(\mathtt{reach}(s0, H, s3)).$$
$$\mathtt{expl}(\mathtt{reach}(s0, H, s3); \mathtt{reach}(s0, H, s4)) \longrightarrow \mathtt{expl}(\mathtt{reach}(s0, H, s4)).$$

- The two productions are not mutually exclusive.

We can *factor* such grammars using Factored Explanation Diagrams (FEDs), which are similar to BDDs.

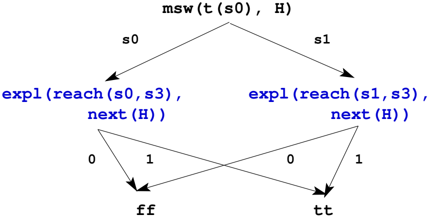
Structure of FEDs

FED is a labeled DAG with

- `tt` and `ff` as leaf nodes
- `msw`(r, h) is an n-ary node if r is a random process with n possible outcomes;

  outgoing edges are labeled with the outcomes.
- `expl`(t, h) is a binary node;

  outgoing edges are labeled 0 and 1.
- If there is an edge from x_1 to x_2, then $x_1 < x_2$ via a specially defined partial order relation.



Operations on FEDs

Boolean operations "$\wedge$" and "$\vee$" can be performed on FEDs along the same line as on BDDs, with one significant change:

- BDD operations are based on a *total* node order.
- We only have a partial node order for FEDs.
- When we recursively push operations down the diagram, we may encounter incomparable nodes.
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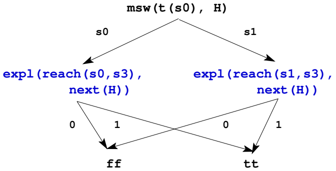
- When we recursively push operations down the diagram, we may encounter incomparable nodes.
- We then generate a placeholder `merge` node, and process merges separately.
- Note that `msw` nodes are always comparable; so a `merge` will involve at least one `expl` node.
- We expand (one of) the `expl` node(s) with its definition, and perform the postponed operation.

FEDs to Equations

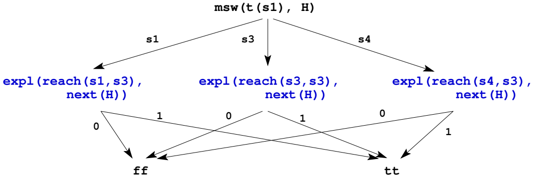
The probability of a set of explanations is computed by generating and solving a set of equations from its FED.

FED for `expl(reach(s0,s3), H)`:



$$
\begin{aligned}
x_0 &= t_{00} * x_0 \\
&\quad + t_{01} * x_1 \\
t_{00} &= 0.5 \\
t_{01} &= 0.3
\end{aligned}
$$

FED for `expl(reach(s1,s3), H)`:



$$
\begin{aligned}
x_1 &= t_{11} * x_1 \\
&\quad + t_{13} * x_3 \\
&\quad + t_{14} * x_4 \\
t_{11} &= 0.4 \\
t_{13} &= 0.1 \\
t_{14} &= 0.5
\end{aligned}
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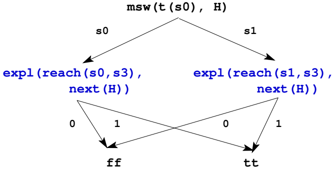

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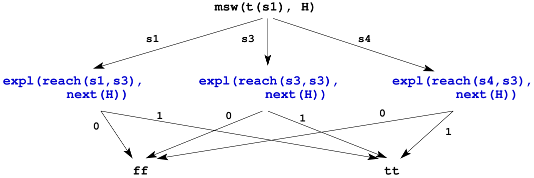

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x_1 &= t_{11} * x_1 \\
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t_{11} &= 0.4 \\
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\end{aligned}
$$

The least solution to these monotone polynomial equations gives the probability of the set of explanations.

Probabilistic Inference for Temporal Queries: Summary

- The set of explanations for a temporal query is conceptually treated as a language defined by a probabilistic grammar.
- This grammar is transformed and materialized as a Factored Explanation Diagram (FED) which ensures that
  - Distinct productions (paths in the diagram) are mutually exclusive.
  - Trials of random variables in a path are independent
- In other words, an FED is a stochastic grammar for he language of explanations.
- Answer probability is computed as the language probability of the grammar: by solving a system of *monotone* polynomial equations.